Potts model for exaggeration of a simple rumor transmitted by recreant rumormongers

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A simple rumor transmitted by recreant rumormongers is considered quantitatively. The simple message contained in the rumor is represented by a simple proposition that has been universally quantified. The operations to change the proposition by rumormongers are established. To describe the rumor's transmission along different channels mathematically, a spin chain is introduced, in which spins represent the operations. The addition of spins is established according to the laws of operations. The result of a rumor's transmission is given by the chain's spin sum. The model, which is favorable for a social prognostication, can determine quantitatively how the social guide and the competition among various opinions affect the exaggeration of the simple rumor transmitted by recreant rumormongers. It proves that the rumor forms Potts-like spin chains in the case with maximum information entropy. The approximate calculation shows that the rumor may be aggrandized little or aggrandized normally, even sometimes catastrophically. Moreover, the exaggeration is greater when the guide is larger and the competition is lower.

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I. INTRODUCTION

At present, phenomena in complex systems are the focus of much attention, with social collective behaviors being especially interesting. One elementary form of the collective behaviors [1–4], is rumor, an unverified story spread from one person to another according to its definition [2]. Many interesting characteristics about it emerge during the course of the transmission of the story. One remarkable one is the uncertainty of the message; that is, the message is often revised continuously by rumormongers. The rumor's exaggeration is also one of these revisions, and could be described quantitatively. Moreover, because an involuntary rumor may probably lead to a political or economic crisis, a quantitative description of a rumor has practical significance.

Up to now, all descriptions of a rumor have been phenomenological [1–4]. As a kind of social processes, a rumor's quantitative description may be provided by the mathematical sociology, which is a subfield of modeling social processes and social structures in sociology [5–8]. Generally, there are multitudinous variables in each mathematical model of social processes, which are characterized by individuals and social groups. Mathematical sociology applies itself to studying the causal relation among these variables, and most of its models describe social processes as stochastic processes conventionally [9–12]. However, there are overfull indefinite parameters in each stochastic process like this, and the degree of difficulty has been increased greatly. Thus we prefer to regard rumor as a complex phenomenon in society and to study it by applying successful methods in physics. In fact, there are many examples of modeling social process physically [13–16]. Usually, a complex system contains three strata the physical, biological, and social [17,18]. Due to its objectivity, a description of complex phenomena in physical systems becomes more quantitative. On the other hand, with the influence that arises due to the subjective motivation of social individuals, phenomena in social stratum are more protean. Therefore, this requires that one adjust every physical result properly when study social phenomena, and that one pay attention to the difference between physical and social phenomena.

From the view popularized in sociology, it is clearly asserted that collective behaviors originate from the motivation and interaction of individuals [1]. Being a kind of whole social behavior, collective behavior's corresponding microscopic behavior is individual behavior, and global social behavior is usually determined by microscopic behavior. We should note that collective behavior is similar to macroscopic behavior in a physical system, and, correspondingly, the social motivation and interaction of individuals are similar to the microscopic motion. Consequently, if provided with a quantitative description, we can determine the relation between these variables by introducing relational variables to describe the rumor's macroscopic and microscopic properties.

Sociologists have explained a rumor's main characteristics phenomenally [1–3]. In order to carry out our discussion, a few relevant characteristics of a rumor are listed as follows.

A rumor's intensity is high when the public is interested in an event with great ambiguity [4]. Each person distorts his or her account by dropping some items and adding his or her own interpretations when a rumor is relayed from one person to another [4]. Social experience suggests that the rumor process can eliminate the most impossible and unreliable content and make its story achieve a high degree of veracity [3].

All this indicates that a rumor often describes an event of public interest. Possessing some uncertainties, this event is spread randomly in public by social individuals. Rumormongers will alter a rumor's description according to their own opinions when transmitting. However, their opinions are correspondingly affected by public viewpoint. Since exaggeration is a rumor's common characteristic, some social mechanisms can restrict unreliable exaggeration automatically.

A stochastic process is the simplest description of a rumor

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process, in which the rumor is only taken as a transmitted thing without any qualitative changes. A rumor's transmission speed is merely figured out by the first-passage time in the stochastic process.

Actually, rumors vary in many aspects. No matter whether it is simple complex, a rumor consists of three basic elements: a transmitted message, rumormongers, and the social effect on the rumor process. With an initial understanding of transmission characteristics, we will depict the simplest rumor quantitatively in a case preserving all three basic elements. The rumor contains a simple proposition, and the rumormongers are so recreant that each of them only dares to change the message slightly. Moreover, what we mainly study is whether or not the proposition can be accepted by society, and how the social guide affects the rumor's transmission. To do so, we have three requirements: quantifying messages, representing the operations which rumormongers use to change the messages, and describing the social effect on rumor transmission clearly.

When quantifying a rumor process, it is necessary to achieve the most abstract and basic representation of a message, which can be provided by the mathematical logic. Mathematical logic studies the fundamental constructions of human thought [9,20], in which the first-order logic is the most fundamental one and is based on simple quantified propositions [19,20]. Each proposition involves three elements: the individual variable that represents the disquisitive subject, the quantified word or quantifier used to restrict individual variables, and the predicate to express the property of the subject.

Simply, a story is a complete message expressed by a quantified simple proposition. When the message is transmitted, its content is revised continuously because each rumormonger adds his or her own interpretation. Correspondingly, the proposition used to describe the message changes continuously. Thus different propositions come at different moments. But these propositions may have the same subject (individual variable), only with different predicates and individual total numbers. Therefore, the distortion of a message means a change of the individual number or a change of the predicate. When a rumor is transmitted, neither the individual number nor which predicate is suitable remains uncertain, which rightly reflects the rumor's ambiguity.

Now we set up an abstract model to demonstrate a rumor's transmission process. Supposing there are L different predicates and the one adopted by the transmitted proposition is among them. The individual number of the proposition may be zero or any positive integer. Therefore, we will adopt a group of propositions below, in which each proposition has the same individual variable as well as arbitrary individual numbers. For the predicate, it can be any one of the L's. Moreover, the proposition expressing a message will vary within the group in this model.

Usually, rumormongers deliver a rumor randomly along various channels. On each channel, each rumormonger receives a proposition, then changes its predicate or individual number and passes it on to the next rumormonger. In our model, we assume that recreant rumormongers add only one to the total individual number of the proposition, or reduce by one according to their own opinions. In order to be more convenient, we introduce the conception of the proposition space.

A rumor's exaggeration is a series of changes cumulated in the proposition's transmission along different channels. We can reckon the individual number of this proposition by following the scent of one channel, and then we can determine the rumor's exaggeration ratio accurately. If the initial individual number of the proposition is n and the number accepted by a certain rumormonger is m, then the rumor's exaggeration ratio is defined as m/n. Otherwise, if we track more channels and select one rumormonger on each channel, the individual number received by these rumormongers amounts to m. After averaging, the exaggeration ration is $\overline{m}/n$.

Sociology indicates that there exists a kind of social guide, which affects the opinion of social individuals on every event of public interest [1–3]. This guide has many sources, such as the authority controlling information, the cultural tradition, popular viewpoints, people's believing in a certain event etc. Among all the opinions concerning the event's property, the social guide leans toward the special one. Generally, we describe the social guide theoretically by the following means. Suppose one of the L's predicates as $F_l(x)$. Compared with the rest, if the social guide ''prefers'' $F_l(x)$, more rumormongers in society will trend toward $F_l(x)$ as well.

In counting the number m, we introduce spin chains to represent a rumor's transmission channels. With N spins, each spin in the spin chain stands for successive operations that recreant rumormongers use to change propositions. The spin sum figures out the cumulative changes of a rumor. Consequently, this model is designed to study the random distribution of rumormongers with different opinions instead of the stochastic transmitting processes of messages. Only in the case of maximum information entropy can the recreant rumormongers' operations form Potts-like spin chains [21–24]. The acceptability exponent σ of a proposition and the guide exponent γ will be defined here, and the spin sum will give the rumor's exaggeration ratio.

When $L>2$, an approximate calculation shows that the rumor will not be exaggerated if $\gamma/\sigma<(1/2)\ln(L-1)+\sqrt{\ln(1+L)/4}$, while it will be aggrandized normally if $(1/2)\ln(L-1)+\sqrt{\ln(1+L)/4}<\gamma/\sigma<[\ln(L^2-1)+1-\ln 4]/2$. However, the rumor is exaggerated dramatically if $\gamma/\sigma>[\ln(L^2-1)+1-\ln 4]/2$. This result shows a mutual restriction between the social guide and the competition among various opinions. The former (social guide) encourages rumor exaggeration while the later (competition) suppresses it.

In accordance with the development of describing models quantitatively in sociology, a quantitative model of social process may be evaluated by two main criteria: empirical adequacy and theoretical adequacy. Obviously, this model is appropriate because all the theoretical results accord with our empirical knowledge about a rumor's transmission in society.

In total, three main processes are involved in this paper: we first propose algebraic and geometric representations of simple propositions with universal quantifiers, and define the proposition space simultaneously (Sec. II). Then we figure out all the operations by which rumormongers change the transmitted proposition; eventually we set up an abstract model of the rumor process (Sec. III). In Sec. IV, we represent a rumor channel with the help of a spin chain, and point out that only the chain with the maximum information entropy is a Potts-like model. The approximate calculation of the spin sum is given in Sec. V. In the Appendixes, we determine laws of addition among spins according to the operations on rumor by rumormongers, and give a detailed calculation of the spin sum.

II. QUANTIFIED SIMPLE PROPOSITION AND PROPOSITION SPACE

A. Simple propositions and proposition group

Not only providing the most fundamental quantitative representation for a message [5,6], mathematical logic has focused on a description of human thought systems with various levels of languages, such as the lower or higher levels. Generally, logic laws on a lower level of language are always feasible on a higher level. Therefore, the lowest level of language is the most fundamental. So as to ensure the universality of this study, a description of a rumor's transmission must be based on the lowest level of language, namely, first-order logic, in which the simplest message is a quantified simple proposition [19,20].

It is convenient to adopt a simple proposition, quantified universally to represent a message when doing the research on the rumor's exaggeration. Each rumor has a certain subject, which represents an interesting event, and remains unchangeable during the transmission. The subject is represented by an individual variable in the quantified proposition, while its property is represented by a predicate. In first-order logic, each individual variable has a domain with finite individuals, and only the individual variables are quantified [19,20]. Supposing the individual variable is x, and its domain includes m individuals. If the predicate is represented as $F(x)$, then a quantified simple proposition with a universal quantifier $(x)_m$ is expressed as follows [19,20]:

$$(x)_m F(x). \tag{1}$$

In the expression above, the bracket (x) represents the universal quantifier ''for all individuals in the domain,'' and the subscription m of the bracket means that the domain of the individual variable contains m individuals.

When transmitted, the message is continuously revised. The revision may be either a change of the individual number m or the predicate. In a group of L different predicates, we denote the predicate set by $\{L\}$, and the lth predicate is expressed as $F_l(x)$. So, when m and l are integers, each quantified simple proposition with the universal quantifier $(x)_m$ can be expressed as follows [19,20]:

$$(x)_m F_l(x). \tag{2}$$

In the exaggeration of a rumor, the individual variable is always invariable during a rumor's transmission but the proposition does change its m or l. We can take the propositions to form a group, assuming $m=0,1,\ldots,\infty$ and $L=1,2,\ldots,L$; then we study all the possible variations of propositions occurring within the group.

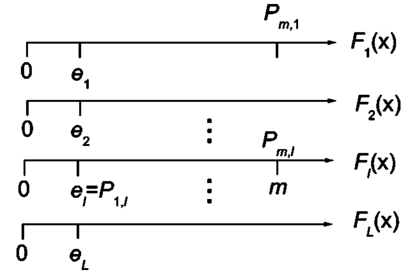

FIG. 1. L positive axes corresponding to L predicates $F_l(x)$. On every axis $e_l=P_{1,l}$ is the unit length that can be used to express the quantified simple proposition as $P_{1,l}=(x)_1F_l(x)$. Every integer m on the lth axis corresponds to the proposition $P_{m,l}=(x)_mF_l(x)$.

We will take a penguin as an example, concerning a group of propositions as follows:

$$\text{all penguins are killed;}$$
$$\text{all penguins are captured;} \tag{3}$$
$$\text{all penguins are released.}$$

Each proposition in the group has the same individual variable x, which here is the penguin, and its domain has m individuals (penguins). The predicate set is $\{3\}$ with three different predicates: $F_1(x)$ is ''are killed,'' $F_2(x)$ is ''are captured,'' and $F_3(x)$ is ''are released.'' Correspondingly, there is a rumor that the transmitted proposition varies in the group. At a certain moment when transmitted, the proposition may be $(x)_3F_3(x)$ (''All three penguins are released'') while it may turn into $(x)_7F_1(x)$ (''All 7 penguins are killed'') at the other moment or some other things like this. Apparently, the proposition has changed m and l during the transmission.

Sociological investigation shows that a rumor's intensity is high when the public is interested in an event with great ambiguity [4]. Clearly, in our discussion of the simplest rumor, the event of interest to society is a group of propositions concerning an individual variable x; the ambiguity usually corresponds to the arbitrariness when we decide m's and l's concrete numbers.

B. Geometric representation of proposition group

A proposition defined by formula (2) is represented by $P_{m,l}$ in the following, in which m stands for an individual number while l for the predicate $F_l(x)$. We can make every predicate $F_l(x)$ correspond to a positive real axis $[0,\infty)$, so that every proposition $P_{m,l}$ corresponds to an integer m on the lth axis. Shown in Fig. 1, there are L positive real axes for the set $\{L\}$, and each integer corresponds to a proposition in the proposition group.

Linked at their original points O, all L axes can form a proposition space. Shown in Fig. 2, the space is a skeleton

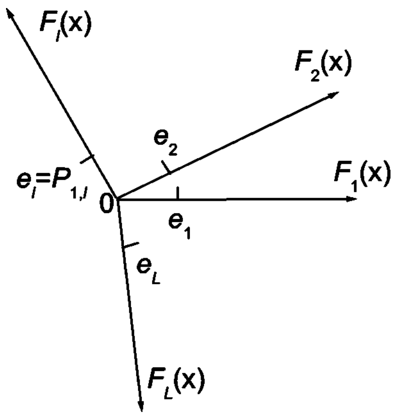

FIG. 2. The proposition space constructed by L axes. It is a skeleton with L branches. In our discussion, only integers on each branch in space correspond to propositions which represent a message.

with L branches. Each proposition, therefore, can be expressed as $P_{m,l}=me_l$ in space.

C. Exaggeration ratio of the rumor

After being put forward by a rumormonger, a proposition is usually spread along various channels during the process of a rumor's transmission among rumormongers. Different channels mean the different experiences of the rumor's proposition. When receiving the rumor at different spots along different channels, an investigator can obtain different exaggeration ratios. It is permitted in theory that we regard all channels as having the same number of rumormongers, and count the exaggeration ratio according to the proposition that is received by the last rumormonger.

In order to be simpler, we choose one channel at first. If the individual number of the initial proposition is n, and that of the proposition received by the last rumormonger is m, we define the exaggeration ratio of the rumor as m/n. As an example, we use a fictitious rumor about a penguin varying in group (3). The original proposition is ''two penguins are captured,'' but the proposition received by the last rumormonger changes into ''eight penguins are released.'' As the rumors increases from two to eight, this means that the original proposition is aggrandized fourfold. Generally speaking, a proposition is put forward by a certain rumormonger initially, and later spread in a network with many different rumor receivers. What we study is the proposition received by the last rumormonger in each channel. If we average the individual number of all propositions received by these rumormongers, we define the exaggeration ratio as $\overline{m}/n$. To be simpler, the individual number of the last proposition we obtain is just the rumor's exaggeration ratio if we suppose the individual number of the initial proposition to be one.

III. OPERATION OF RUMORMONGERS ON A PROPOSITION

In our model, each rumormonger has his own opinion to assert as a specific predicate $F_l(x)$ when a message is transmitted through various channels. This changes the proposition received according to his opinion, and passes it on to the rumormonger. Based on the assumption of a recreant rumormonger, proposition $P_{m,k}$ can be changed $P_{m+1,k}$ if $k=l$, which means that the proposition and the rumormonger have the same predicate. Otherwise, it will be changed into $P_{m-1,k}$ when $m\neq 0$ or $P_{1,l}$ when $m=0$ if we suppose that $k\neq l$. Having asserted the predicate $F_2(x)$ (''are captured''), a rumormonger will send proposition $P_{m+1,2}$ (''all $m+1$ penguins are captured'') to the next rumormonger if he receives proposition $P_{m,2}$ (''all m penguins are captured''), while he will send $P_{m-1,1}$ (''all $m-1$ penguins are killed'') or $P_{m-1,3}$ (''all $m-1$ penguins are released'') to the next rumormonger if he receives $P_{m,1}$ (''all m penguins are killed'') or $P_{m,3}$ (''all m penguins are released''), respectively.

Now we turn to consider the channel with $N+1$ rumormongers. The operation, by which a recreant rumormonger changes the proposition at the nth site, will be expressed as $O_{n,l}$ $(n=0,1,\ldots,N)$ if he asserts the predicate $F_l(x)$. Its mathematical expression is

$$O_{n,l}P_{m,k}=\begin{cases} P_{m+1,k}, & l=k \\ P_{m-1,k}, & l\neq k,\ m\neq 0 \\ P_{1,l}, & l\neq k,\ m=0 \end{cases}. \tag{4}$$

If the proposition received by the rumormonger at the end of this channel is $P_{m,l}$, we can average m for all possible channels. Moreover, we must know the distribution of rumormongers along each channel before obtaining $\overline{m}$ (the average m).

IV. POTTS MODEL FOR RUMOR

A. Rumor expressed by a spin chain

The mathematical representation of the operations by which rumormongers change propositions was given in Sec. III. In this section, we will point out a rumor' transmission channel with the help of a spin chain, and determine the changes of the propositions cumulated in transmission by the addition of spins.

We come to the case of $L=2$, which has the corresponding predicate set $\{2\}$. With only two different predicates $F_1(x)$ and $F_2(x)$, set $\{2\}$ has two branches of proposition space corresponding to these predicates respectively. The proposition space becomes a real axis if we assume $e_1=1$ and $e_2=-1$. Let us look at the rumor transmitted along a certain channel. The initial rumormonger $O_{0,l}$ proposes an original proposition $P_{1,1}$ (or $P_{1,2}$), then the successive rumormonger $O_{n,l}$ changes the proposition and sends it to the next rumormonger according to formula (4). This formula shows that the operation of a rumormonger is to add 1 to i or subtract 1 from it in the proposition $P_{i,j}$.

A spin chain can express the operations of rumormongers step by step. For a chain with $N+1$ spins S_n $(n=0,1,\ldots N)$, the subscription n of S_n denotes the nth site of the spin chain. Each rumormonger $O_{n,l}$ corresponds to one spin S_n, and takes the value e_l $(l=1$, and 2) due to the predicate claimed.

Operation (4) should correspond to the addition of spins, as shown in the following. Suppose that the original proposition sent out by the initial rumormonger $O_{0,l}$ is $P_{1,1}$; this can be described as $S_0=e_1=1$ in the spin chain. If the second rumormonger is $O_{1,l}$, and the rumor described as $S_1=e_l$, according to $l=1$ or $l=2$, the proposition sent out by the second rumormonger should be $P_{2,1}$ or $P_{0,1}$ after operation (4). Therefore, the number of proposition individuals (2 or 0) sent out is given by the addition of spins S_0+S_1 as follows:

$$S_0+S_1=e_1+e_l=\begin{cases}1+1=2 & \text{for } l=1\\ 1+(-1)=0 & \text{for } l=2.\end{cases} \tag{5}$$

By induction, the proposition received by the $(n+1)$th rumormonger is $P_{i,j}$ with $i=|\Sigma_{q=0}^{n}S_q|$, while that one received by the last rumormonger is $P_{m,k}$ with $m=|\Sigma_{q=0}^{N}S_q|$. In these expressions, the sign of the absolute value shows that the individual number of a proposition is a natural number.

The system is a Potts-like spin chain model, with every spin S_n having L components when $L>2$ [7]. If the proposition received by the $(n+1)$th rumormonger $O_{n,l}$ is $P_{i,k}=ie_k$, shown as in Eq. (4), the proposition sent out by him is $P_{i+1,k}$ or $P_{i-1,k}$. Correspondingly, the addition of spins should be defined as

$$ie_k+e_l=\begin{cases}(i+1)e_k & \text{for } l=k\\ (i-1)e_k & \text{for } l\neq k\end{cases} \quad \text{for } i\neq 0 \tag{6}$$

and

$$0+e_l=e_l \quad \text{for } i=0. \tag{7}$$

Since all details of the operations of spins are given in Appendix A, only some results are induced directly. We can also define the absolute value of me_k as $|me_k|=m$, and suppose the addition of a polynomial to be carried out successively. So, it is quite easy to understand the rationality of the addition. Similar to the case of $L=2$, the proposition $P_{m,k}$ received by the last rumormonger is expressed $m=|\Sigma_{n=0}^{N}S_n|$.

B. Statistical models without social guide

The view popularized in sociology asserts that collective behavior originates from the motivation and interaction of individuals [1]. Collective behavior is a kind of whole social behavior, and its corresponding microscopic behavior in society is the motivation and interaction of individuals. As we already described above, social behaviors and physical systems have much in common. For example, collective behavior is similar to the macroscopic property in a physical system, and the motivation and interaction of individuals are similar to the movement and interaction of physical subsystems (particles).

For the process of a rumor's transmission described by a spin chain in Sec. III, the group of rumormongers represented by a spin chain is a macroscopic system, and each rumormonger $O_{n,l}$, represented by a spin S_n, is a subsystem. In a physical system, macroscopic behaviors are determined by the behaviors of subsystems. Obviously, being a kind of macroscopic behavior of the rumor process, the exaggeration ratio of a rumor is determined by the microscopic behaviors of all subsystems (the spins).

On the other hand, the macroscopic behavior of a physical system restricts the microscopic behaviors of all its subsystems as well as determining the probability of microscopic configurations [22–24]. Similarly, there are some macroscopic variables that determine the statistical properties of a rumor's channels. One basic property of a rumor is that most improbable and unreliable accounts of a rumor are eliminated during transmission. This means that we should consider the acceptability of a proposition to a group of rumormongers, and introduce an acceptability exponent to prescribe the probability of spin configurations. At the same time, another factor affecting the configuration probability is the social guide, which induces more rumormongers to believe a certain assertion. The guide exponent will also be introduced.

An original rumor is always spread along all possible channels among rumormongers. The propositions $P_{m,k}$ received by the last rumormongers in different channels will have different values of m and k. Obviously, m and k are determined by the opinions of all rumormongers. Therefore, it is only possible for us to predict average values of m and k.

In a spin chain, each spin S_q is the discrete variable and takes a value in the set $(e_1,e_2,\ldots,e_L)$. The set of S_q on a chain forms a stochastic series $(S_0,\ldots,S_q,\ldots,S_N)$. Denoting the probability of the series $(S_0,\ldots,S_q,\ldots,S_N)$ with the value $(e_l,\ldots,)$ as $p(S_0,\ldots,S_q,\ldots,S_N)$, we can write the information entropy of the series as

$$S=-\sum_{\{S_q\}} p(S_0,\ldots,S_q,\ldots,S_N)\ln p(S_0,\ldots,S_q,\ldots,S_N), \tag{8}$$

where the subscript $\{S_q\}$ is the sum over all configurations of the series. The normalization condition of the probability is

$$\sum_{\{S_q\}} p(S_0,\ldots,S_q,\ldots,S_N)=1. \tag{9}$$

In this section, we study a rumor without a social guide. Thus there is no dominant opinion concerning the event of public interest. All opinions asserting different predicates will appear with the same probability. According to this assumption, all components of spins at the chain will also appear with the same probability. Thus the probability of the sum $\Sigma_{q=0}^{N}S_q=me_k$ has nothing to do with the subscription k, and the average value of m is only limited by the social acceptability of the rumor, namely,

$$\sum_{\{S_q\}} mp(S_0,\ldots,S_q,\ldots,S_N)=\overline{m}. \tag{10}$$

We consider the most random case, in which the information entropy [Eq. (8)] takes an extreme value with a given $\overline{m}$, and the normalization condition [Eq. (9)]. Introducing the Lagrangian multipliers $1/\sigma$ and λ, we obtain the expression of the probability as follows:

$$p(S_0,\ldots,S_q,\ldots,S_N)=Q(\sigma)^{-1}e^{-m/\sigma}. \tag{11}$$

This result has a discrepancy of $O(1/N)$ according to the standard statistical method in physics [22–24]. In Eq. (11), we have $Q(\sigma)=\Sigma_{\{S_q\}}e^{-m/\sigma}$. This model is a Potts-like chain of spins with L components [21]. Inserting Eq. (11) into Eq. (10), we obtain the following equation; as

$$\overline{m}=\overline{m}(\sigma). \tag{12}$$

This equation determines the multiplier σ by a given $\overline{m}$. Conversely, the multiplier σ also determines the average value $\overline{m}$. The average value $\overline{m}$ determines the exaggeration ratio of the rumor, and manifests the social effect on the rumor process. For a fixed m, Eq. (11) shows that, the larger σ is, the larger the probability of a configuration. This means that society may accept a larger exaggeration ratio of the rumor if σ is larger. Therefore, we can call the multiplier σ the acceptability exponent of the rumor according to our definition.

C. Statistical models with the social guide

In our model, a social guide ‘‘prefers’’ some predicate $F_l(x)$ in the group $\{L\}$. With the effect of this guide, more rumormongers will assert the same predicate as the guiding opinion. If the social guide prefers the nth predicate, the probability of the spin sum $\Sigma_{q=0}^{N}S_q=me_n$ will be larger than that of the other sum $\Sigma_{q=0}^{N}S_q=me_k$ $(k\neq n)$. This case can be expressed as follows:

$$p(P_{m,n})/p(P_{m,k})=\exp[(\gamma+1)m/\sigma],\quad \gamma>-1. \tag{13}$$

In this equation, the factor $\exp[(\gamma+1)m/\sigma]$ shows the ratios of the probability with and without social guides. The larger the factor, the larger the probability of the rumormongers asserting the nth predicate. Thereby, the factor $\gamma+1$ can be considered the guide exponent.

In the case of a social guide, the probability of the series $(S_0,\ldots,S_q,\ldots,S_N)$ is

$$p(S_0,\ldots,S_q,\ldots,S_N)=Q(m)^{-1}\exp\left\{-\left[1-\delta\left(\sum_{q=0}^{N}S_q,n\right)(\gamma+1)\right]m/\sigma\right\}. \tag{14}$$

Moreover, we have also defined

$$\delta\left(\sum_{q=0}^{N}S_q,n\right)=\begin{cases}1 & \text{for } k=n\\ 0 & \text{for } k\neq n,\end{cases}\quad \sum_{q=0}^{N}S_q=me_k. \tag{15}$$

Naturally, the model of a rumor with a social guide should turn into one without a social guide if the guide disappears.

V. RESULTS OF THE APPROXIMATE CALCULATION

When $L>2$, the probability sum can be represented approximately as (Appendix B)

$$Q_m=\frac{(L^2-1)^{N/2}(L-1)(L-2)}{L}\times\left\{L+\sum_{m=1}^{N}[(L-1)e^{-am}+e^{bm}]\times\sum_{l=m}^{N}\sqrt{\frac{2}{\pi l}}\exp\left[-\frac{m^2}{2l}-\frac{l}{2}\ln\left(\frac{L+1}{4}\right)\right]\right\}, \tag{16}$$

in which

$$a=(1/2)\ln(L-1)+1/\sigma,\quad b=\gamma/\sigma-(1/2)\ln(L-1). \tag{17}$$

The average $\overline{m}$ is determined by the following factor in formula (16):

$$\exp\left[bm-\frac{m^2}{2l}-\frac{l}{2}\ln\left(\frac{L+1}{4}\right)\right]. \tag{18}$$

Its exponent can be written as

$$\left[b-\sqrt{\ln\left(\frac{L+1}{4}\right)}\right]m-\left[\frac{m}{\sqrt{2l}}-\sqrt{\frac{l}{2}\ln\left(\frac{L+1}{4}\right)}\right]^2, \tag{19}$$

or

$$\left[b^2-\ln\left(\frac{L+1}{4}\right)\right](l/2)-(m-bl)^2/2l. \tag{20}$$

Apparently, there are two special values b_1 and b_2, which are shown, respectively, as

$$b_1=\sqrt{\ln\left(\frac{L+1}{4}\right)} \tag{21}$$

and

$$b_2=\left[1+\ln\left(\frac{L+1}{4}\right)\right]\Big/2. \tag{22}$$

There exist three cases.

When $b<b_1$, only the lowest l and m provide the effective contribution to Q_m. So we can obtain

$$\sum_{\{S_q\}}mp(m)\approx 1. \tag{23}$$

This result means that the rumor has not been aggrandized if $\gamma/\sigma<(1/2)\ln(L-1)+\sqrt{\ln(1+L)/4}$.

When $b>b$, the maximum term in Q_m is that $l=m=N$. So we can obtain

$$\sum_{\{S_q\}}mp(m)\approx N. \tag{24}$$

This result means that the rumor is aggrandized dramatically if $\gamma/\sigma > \{1+\ln[(L^2-1)/4]\}/2$.

When $b_1 < b < b_2$, we can obtain

$$1 < \sum_{\{S_q\}} mp(m) < N. \tag{25}$$

This result means that the rumor is aggrandized normally if $\{1+\ln[(L^2-1)/4]\}/2 > \gamma/\sigma > (1/2)\ln(L-1)+\sqrt{\ln(1+L)/4}$.

From the discussion above, we find that the exaggeration is greater if the rumor has a larger social guide and a lower ambiguity.

VI. SUMMARY

We consider a rumor's transmission quantitatively. We propose algebraic and geometrical representations of simple propositions with a universal quantifier, and define the proposition space simultaneously. Then we figure out all operations by which rumormongers change the transmitted proposition; eventually we set up an abstract model of rumor process. We introduce the acceptability exponent σ of a proposition and the guide exponent γ to describe the effect on the rumor process by society. We represent a rumor channel with the help of a spin chain, and point out that only the chain with maximum information entropy is a Potts-like model. The spin sum will give the rumor's exaggeration ratio. The approximate calculation of the spin sum is given. The result shows the different exaggerations as follows. The rumor will not be exaggerated if $\gamma/\sigma < (1/2)\ln(L-1) + \sqrt{\ln(1+L)/4}$, while it will be aggrandized dramatically if $\gamma/\sigma > \{1+\ln[(L^2-1)/4]\}/2$. However, the rumor is exaggerated normally if $\{1+\ln[(L^2-1)/4]\}/2 > \gamma/\sigma > (1/2)\ln(L-1) + \sqrt{\ln(1+L)/4}$. All the results accord with our empirical knowledge about a rumor's transmission in society.

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APPENDIX A: ADDITION OF SPINS

According to the laws of operations which change propositions by rumormongers [Eq. (4)], the addition of S_n is defined as follows:

$$ie_k + e_l = \begin{cases} (i+1)e_k & \text{for } l=k \\ (i-1)e_k & \text{for } l\neq k \end{cases} \quad \text{for } i\neq 0, \tag{A1}$$

$$0 + e_l = e_l \quad \text{for } i=0. \tag{A2}$$

The addition of a polynomial is supposed to be carried out successively.

In the example when $L=3$, the addition of e_l has neither an associative law nor a commutative law; e_1, e_2, and e_3 are involved in it. One polynomial is $e_1 + e_2 + e_3$, and the addition should be carried out successively as follows:

$$e_1+e_2+e_3 = (e_1+e_2)+e_3 = 0+e_3 = e_3. \tag{A3}$$

If we combine e_2 and e_3 at first, the addition will become

$$e_1+(e_2+e_3) = e_1+0 = e_1. \tag{A4}$$

Equations (A3) and (A4) show that the addition defined here does not satisfy the associative law. Another polynomial is $e_1+e_3+e_2$, in which the order of e_2 and e_3 is commutated compared with the former one. We can obtain

$$e_1+e_3+e_2 = (e_1+e_3)+e_2 = 0+e_2 = e_2. \tag{A5}$$

This equation shows that addition also does not satisfy the commutative law. So the calculation of spin sum will be especially difficult due to the property described above. When $L>3$, we'll arrive at the same conclusion.

APPENDIX B: PROOF OF EQ. (16)

What we are interested in is the case of $L>2$. Its probability sum is

$$Q_m = \sum_{\{S_q\}} p(S_1, \ldots, S_q, \ldots, S_N) = A_m[(L-1)e^{-m/\sigma} + e^{\gamma m/\sigma}], \tag{B1}$$

where A_m is a configuration number corresponding to the spin sum $|\Sigma_{q=1}^N S_q| = m$. The coefficient $L-1$ of the factor $e^{-m/\sigma}$ is introduced because of $L-1$ predicates in the predicate set $\{L\}$, which are different from the predicates asserted by the social guide.

The calculations of the spin sum $\Sigma_{q=1}^N S_q$ and the configuration number A_m are the key to give Q_m. The addition of spins [Eq. (A1)] can be rewritten as follows:

$$\sum_{q=1}^{n+1} S_q = ke_r + S_{n+1} = \begin{cases} (k+1)e_r, & S_{n+1}=e_r \\ (k-1)e_r, & S_{n+1}\neq e_r. \end{cases} \tag{B2}$$

The spin sum ke_r is still proposition $P_{k,r}$, which corresponds to the point ke_r on the axis $F_r(x)$ in the proposition space. The above law means that the addition of spins corresponds to the movement of the point expressing the proposition from ke_r to $(k+1)e_r$ or $(k-1)e_r$. Therefore, the addition of N spins corresponds to the N-step walk of a point representing a proposition in proposition space. The configuration number A_m is just the number of different paths of the N-step walk.

The proposition space is a skeleton with L branches (axes). Each N-step walk in the space proceeds as follows: A point representing a proposition is initially at a position e_r, which means that the proposition has only one individual. The point jumps forward or backward one unit each step. After some instances of random backward and forward steps from original point O along the branches, it arrives at a position me_r. Let the number A_m be the total path number of an N-step walk from the original point O to the point me_r, $(r=1,2,\ldots,L)$ on the axis $F_r(x)$. The number A_m can be counted as follows. The N-step walk is divided into two sections. One of them is a $2n$-step transition from O point to O point with a number of different paths A_{2n}. Another is the last l-step walk on two axes $F_r(x)$ and $F_\nu(x)$ with the number of different paths A_l. Therefore, we have

$$A_m=\sum_{n=0}^{[(N-m)/2]} A_{2n}A_l, \tag{B3}$$

where $l=N-2n$, and $[(N-m)/2]$ is the maximum integer less than or equal to $(N-m)/2$. The numbers A_{2n} and A_l will be calculated below.

In the set $\{L\}$, only one predicate is the same as that of the proposition me_r, but $L-1$ predicates are different. Thus, if we use the walk to describe the addition of spins, the probability is $1/L$ for the translation $ke_r\rightarrow(k+1)e_r$, but is $(L-1)/L$ for the translation $ke_r\rightarrow(k-1)e_r$.

Each $2n$-step walk can be divided again into t smaller sections $t=1,\ldots,n$. Each smaller section is a translation on one axis in the proposition space. It starts from the O point, arrives at some point on the axis different from the O point, and then returns to the point O again. Thus there are three factors for determining A_{2n}. One factor is the number of different methods cutting $2n$ steps into t smaller sections containing even steps, and this $C_n^t=n!/t!(n-t)!$.

The translation of the first $t-1$ smaller sections can be on all L axes, which gives the factors L^{t-1}. If the last l-step walk is on the axes $F_r(x)$ and $F_\nu(x)$, the tth section can be on all $L-2$ axes except the axes $F_r(x)$ and $F_\nu(x)$. Here $F_\nu(x)$ may possibly belong to the $L-1$ axis, but not $F_r(x)$.

The calculation of the number of different paths of each smaller section on a certain axis is very complicated. Here we will apply an approximation in which only one of the different paths is taken into account.

Half of the $2n$ steps must point in the O point direction; the n steps that remain point in the opposite direction. So a factor $(L-1)^n$ should be involved in the configuration number. Thus we can express A_{2n} as follows:

$$A_{2n}=(L-1)^n\sum_{t=1}^{n} C_n^t L^{t-1}(L-1)(L-2)$$
$$=\frac{(L^2-1)^n(L-1)(L-2)}{L}. \tag{B4}$$

The last l-step transition is just one-dimensional random walk, so

$$A_l=\frac{l!}{\left(\frac{l-m}{2}\right)!\left(\frac{l+m}{2}\right)!}(L-1)^{(l-m)/2}, \tag{B5}$$

where the factor $(L-1)^{(l-m)/2}$ means that there are $(l-m)/2$ in l steps pointing in the O point direction. Using Sterling's formula under the assumption of smaller m, we can rewrite the above formula as

$$A_l=\sqrt{\frac{2}{\pi l}}e^{-m^2/2l}(L-1)^{(l-m)/2}2^l,\quad \text{for } l\neq 0 \quad\text{and}\quad m\neq 0, \tag{B6}$$

$$A_l=1 \quad \text{for } l=0 \quad\text{and}\quad m=0. \tag{B7}$$

Inserting the formulas (B4) and (B6) into formula (B1), we obtain the probability sum

$$Q_m=(L^2-1)^{N/2}(L-1)(L-2)\times\left\{1+\frac{\sum_{m=1}^{N}[(L-1)e^{-am}+e^{bm}]}{L}\times\sum_{n=0}^{(N-m)/2}\sqrt{\frac{2}{\pi(N-2n)}}\times e^{-[m^2/2(N-2n)]-[(N-2n)/2]\ln[(L+1)/4]}\right\}, \tag{B8}$$

where $a=1/\sigma+(1/2)\ln(L-1)$ and $b=\gamma/\sigma-(1/2)\ln(L-1)$. The above formula can be rewritten as

$$Q_m=(L^2-1)^{N/2}(L-1)(L-2)\times\left\{1+\frac{1}{L}\sum_{m=1}^{N}[(L-1)e^{-am}+e^{bm}]\times\sum_{n=m}^{N}\sqrt{\frac{2}{\pi l}}e^{-(m^2/2l)-(l/2)\ln[(L+1)/4]}\right\}. \tag{B9}$$

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